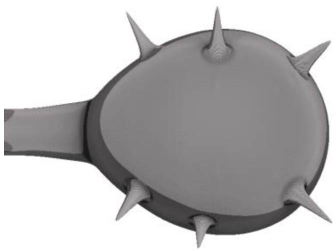
AdS/CFT for CFT/AdS



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Caltech conference on “$\mathcal{N}$=4 SYM theory, 35 years after”,

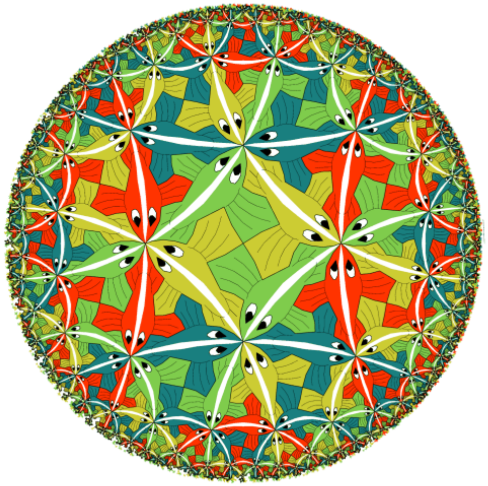
March 30, 2012

Based on : OA, Marolf, Rangamani, arXiv:1011.6144;
OA, Berdichevsky, Berkooz, Shamir, arXiv:1106.1870;
work in progress

Motivations

- In the second half of the history of the 4d $\mathcal{N}=4$ SYM theory, since 1994, we have learned a lot about the strong coupling behavior of this theory, using the tools of S-duality, the AdS/CFT correspondence, and integrability. These taught us about the theory on $R^{3,1}$ and on $S^3 \times R$.
- Can we learn more by putting the theory on other space-times ? Easiest to study space-times that have a large symmetry group and preserve a lot of supersymmetry, so we will consider this theory on the maximally symmetric space AdS_4.

- In global coordinates AdS_4 behaves like a box in many ways; free fields on this space all have energies $\#/L_{AdS}$ with positive coefficients. So, anti-de Sitter space provides an IR cutoff in a maximally symmetric fashion ($SO(3,2)$ isometry), and is useful for studying strongly coupled theories without worrying about IR divergences. Can preserve 16 supercharges (3d $\mathcal{N}=4$ superconformal algebra) for appropriate boundary conditions.
- Also relevant for AdS/CFT; strong coupling in the bulk.

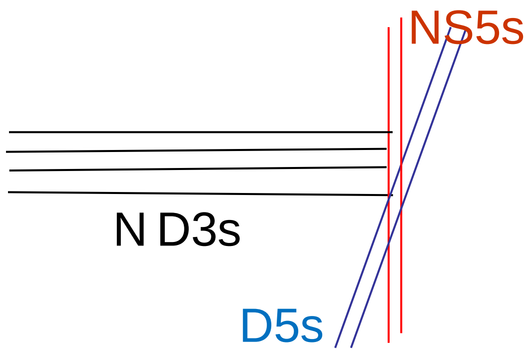


- To study the theory on AdS_4, it is useful to note that AdS_4 is conformally equivalent to the half-space (t,x,y;z>0). This is evident in Poincare coordinates,

$$ds^2 = L_{AdS}^2 (dt^2 - dx^2 - dy^2 - dz^2)/z^2.$$

- Thus, if we choose boundary conditions that preserve the conformal symmetry, the theories on these two spaces are equivalent, and will have the same S-duals and AdS/CFT-duals. Of course the interesting questions to ask about the two cases may be different, but have a simple translation.
- I will discuss the 4d $\mathcal{N}$=4 SYM theory on these two space-times, going freely between the two languages as convenient.

- The half-space picture is particularly useful because the 4d $\mathcal{N}$=4 SU(N) SYM theory on a half-space can be realized (for specific boundary conditions) as the low-energy limit of a brane configuration, involving N D3-branes ending on 5-branes. This was used by Gaiotto and Witten to help classify the possible supersymmetric boundary conditions, and we will see it is useful for finding the AdS/CFT duals as well.



Outline

1) Motivations (done)
2) Review of boundary conditions for 4d $\mathcal{N}$=4 supersymmetric Yang-Mills (SYM)
3) Dual gravitational backgrounds :
   (a) Orientifolds and orbifolds;
   (b) 3-branes ending on 5-branes
4) Application : confinement in $\mathcal{N}$=4 SYM !?

Boundary conditions for $\mathcal{N}$=4 SYM

- The behavior of the $\mathcal{N}=4$ SYM theory on a space with a boundary, and in particular its string theory dual, depends on the choice of boundary conditions.
- The theory includes a vector field, four fermions, and six conformally coupled scalars with $m^2=-2/L_{AdS}^2$
- For $U(1)$ $\mathcal{N}=4$ SYM (free fields) each of these fields has two (linear) choices of boundary conditions preserving 3d conformal symmetry. For the scalar fields this is the standard story in this mass range; near the boundary $\phi(z)\sim az+bz^2+\ldots$, and have either Dirichlet $a=0$ ($\Delta=2$ in CFT_3) or Neumann $b=0$ ($\Delta=1$). Similar story for massless fermions.

- For gauge fields on AdS_4, there is also a similar situation. The standard boundary condition is Dirichlet $A_i(z=0)=0$ (in $A_z=0$ gauge), giving a global symmetry in AdS_4/CFT_3 (corresponding to constant gauge transformations that do not vanish at the boundary). But can also have Neumann, $d_z A_i(z=0)=0$, corresponding to gauging the global symmetry in 3d (integrating over $A_i(z=0)$). In the first case $F_{ij}=0$, and in the second $F_{iz}=0$, so the two cases are related by S-duality. New U(1) global symmetry from $J={}^*F_{3d}$.
- These boundary conditions map to the usual Dirichlet or Neumann boundary conditions on half-space.

- Cannot preserve all SUSY, but can preserve half (3d $\mathcal{N}$=4 superconformal algebra). Under this symmetry have vector multiplet (A_i, Y_a, fermions) and hypermultiplet (A_z, X_b, fermions), and must choose same boundary conditions in each multiplet. Have two choices : Dirichlet for vector, Neumann for hyper or Dirichlet for hyper, Neumann for vector. Exchanged by S-duality. First is realized by D3-brane ending on D5-brane, second by D3-brane ending on NS5-brane.
- What changes in non-Abelian case (G=U(N)) ? One difference is that boundary conditions could break the gauge group. For instance, can choose Neumann b.c. for subgroup H in G (gauge part of global symmetry on boundary).

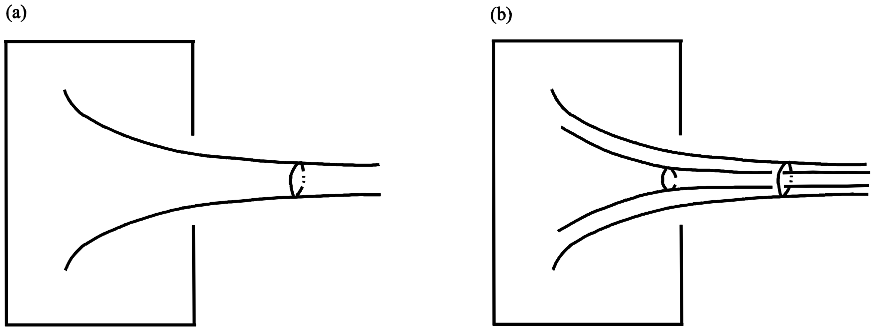
- Naively expect that the two simplest choices are still S-dual, but this cannot be true : for instance, Dirichlet for vector has $U(N)$ global symmetry, while Neumann has just $U(1)$ global symmetry.
- Have two extra possibilities (classified by Gaiotto-Witten for half-space) :

1) Can couple to 3d $\mathcal{N}=4$ SCFT on the boundary.
2) Have many different supersymmetric vacua, involving non-zero VEVs for scalar fields; SUSY is preserved on AdS_4 whenever $W'+K'/L_{AdS}=0$, and this gives (classically) $X_i= iL_{AdS}\varepsilon_{ijk}[X_j,X_k]$. So, can have constant VEVs $X_i = \tau_i/L_{AdS}$, where τ_i is any N-dimensional representation of $SU(2)$.

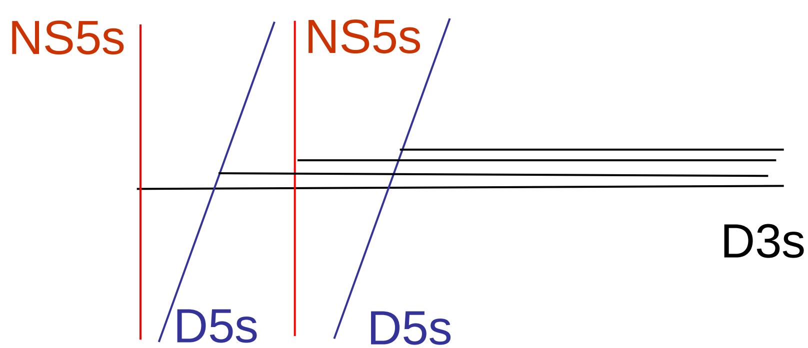
- Both are realized in simple brane constructions.

- N D3-branes ending on a single D5-brane should have just a U(1) global symmetry : have Dirichlet for vector but X_i goes as N-dimensional representation of SU(2), SU(N) broken. ("Fuzzy funnel".) S-dual to Neumann, realized by a single NS5-brane.
- N D3-branes ending on N D5-branes have U(N) global symmetry : $X_i=0$. S-dual to N D3-branes ending on N NS5-branes, but there have non-trivial 3d $\mathcal{N}=4$ SCFT at the boundary (with U(N) global symmetry).
- Non-trivial SCFTs arise whenever D3-branes end on more than one NS5-brane. This can be seen by slightly separating the NS5-branes, giving some 3d $\mathcal{N}=4$ gauge theory on the D3-branes stretched between them, that flows to the SCFT in the IR.

- Generally each 5-brane can have M D3-branes ending on it. In the quantum theory this leads (on the half-space) to a bending of this 5-brane by an amount proportional to M. If we have k 5-branes with the same number of D3-branes ending on each, we have a U(k) global symmetry.



- For D3-branes ending both on D5-branes and on NS5-branes, also have non-trivial 3d $\mathcal{N}$=4 SCFT at the boundary, but the full story is more complicated, since D3-branes are created when shift 5-branes. Configuration characterized by linking numbers – e.g. for each 5-brane, how many D3-branes end on it from one side, minus the number of D3-branes ending from other side, plus the number of 5-branes of opposite type on the other side. This number is preserved by moving the 5-branes around; controls bending.



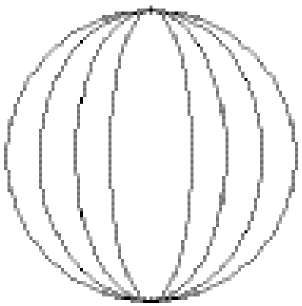
Strongly coupled behaviour

- How does the 4d $\mathcal{N}$=4 SYM theory, with the various boundary conditions we discussed, behave at strong coupling ?
- At very strong coupling, $g_{YM} >> 1$, can use S-duality in the bulk. The action of S-duality on the boundary conditions was understood by Gaiotto+Witten for the half-space, and is the same on AdS_4. In the brane construction one just exchanges D5-branes with NS5-branes.
- Can we find a gravitational dual description that will be useful for large N and large 't Hooft coupling $g_{YM}^2 N$? For which boundary conditions ?

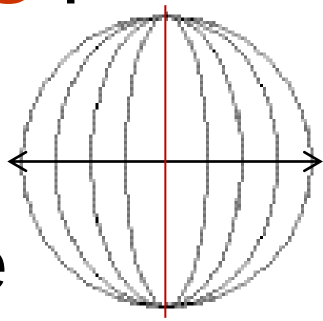
- For some boundary conditions we can have large global symmetry groups (up to U(N)), that should map to gauge symmetries in the bulk. So, not just pure (super)gravity. The brane construction suggests we should have NS5-branes and/or D5-branes in the bulk carrying these gauge symmetries.
- One obvious problem is that the (conformal) boundary of the gravitational space-time should be the space that the field theory lives in, AdS_4. But how can a boundary have a boundary ?
- Let’s look for appropriate string theory solutions and see how this problem is resolved…

Simple solutions

- We expect the bulk to include an approximately $AdS_5 \times S^5$ region that is dual to the field theory away from its boundary; but the boundary of the gravitational theory should be AdS_4 instead of $R^{3,1}$.
- In fact, AdS_5 has an AdS_4 slicing (used e.g. for Randall-Sundrum). However, in this slicing the boundary of AdS_5 is mapped to <u>two</u> copies of AdS_4, connected at their boundary, so it is not quite what we want.
- To get one copy of AdS_4, we need to identify the two boundaries.



- The simplest way to get a holographic dual of the SYM theory on a single AdS_4 is to perform an orbifold / orientifold that identifies the two copies of AdS_4. This can preserve half of the SUSY for an orbifold / orientifold 5-plane wrapping $AdS_4 \times S^2$.
- This gives simple solutions that are naturally identified with the field theory on D3-branes intersecting orbifold / orientifold 5-planes. The corresponding boundary conditions were analyzed by Gaiotto+Witten, and they depend on the precise choice of orbifold / orientifold. For $O5^-$ and $O5^+$ orientifold planes, one has Neumann boundary conditions for USp(N) and SO(N) subgroups of U(N), respectively (and Dirichlet for rest).



- This construction gives us string theory (not purely gravitational) duals for a large class of boundary conditions, as orbifolds / orientifolds of type IIB string theory on AdS_5 x S^5. "Boundary of boundary" is realized by identification. What about the boundary conditions coming from D3-branes ending on 5-branes ?

Gravitational solutions for D3-branes ending on 5-branes

- To find the gravity solutions for D3-branes ending on 5-branes, recall that the most general solutions of type IIB supergravity with 3d $\mathcal{N}$=4 superconformal (OSp(4|4)) symmetry were already found in 2007 by D'Hoker, Estes and Gutperle. They were mainly interested in D3-branes intersecting 5-branes, or in "Janus solutions" in which the coupling constant of $\mathcal{N}$=4 SYM changes as one crosses some 3d defect. However, precisely the same symmetries arise for D3-branes ending on 5-branes, so solutions (at least for some boundary conditions) should be included in their general classification.

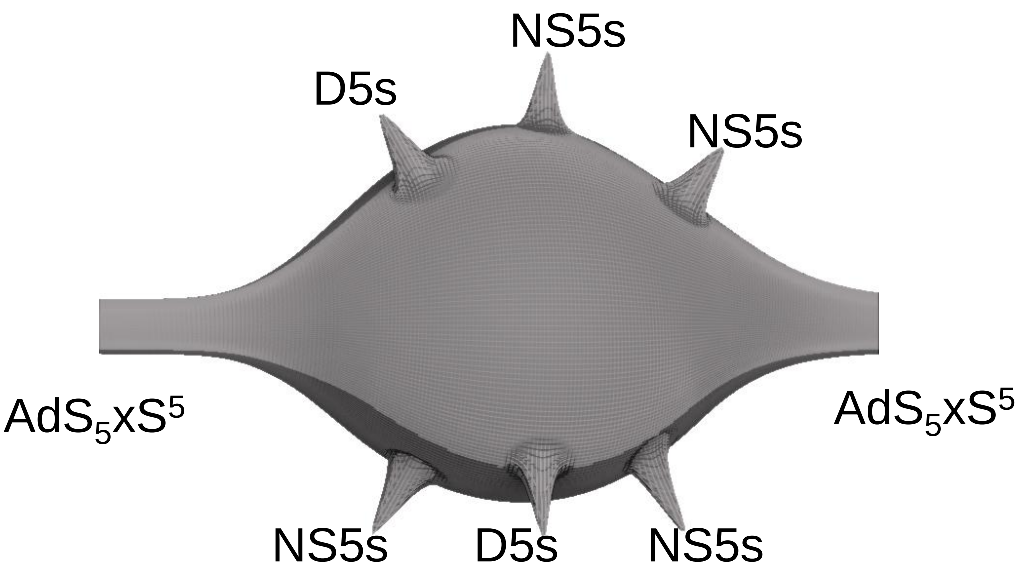
- Review of their solutions : the symmetries imply that the geometry should have an AdS_4 factor and two S^2 factors. So, the type IIB geometry is a warped product of these with a Riemann surface Σ,

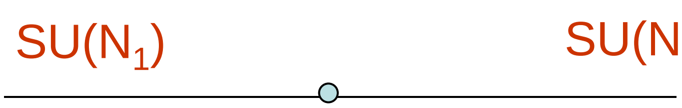
$$ds^2 = f_4^2(w)ds^2_{AdS_4} + f_1^2(w)ds^2_{S_1^2} + f_2^2(w)ds^2_{S_2^2} + ds^2_\Sigma.$$

  Also have some 3-forms (RR and NS-NS) on the S^2's (times a 1-form in Σ), a 5-form on both S^2's (times a 1-form in Σ), and a dilaton $\Phi(w)$. (no axion)

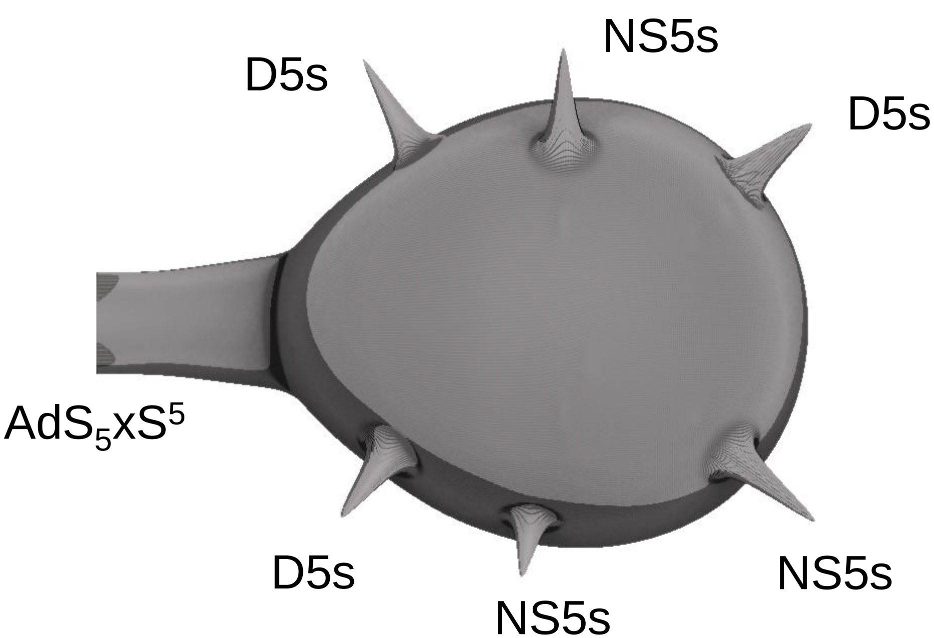
- D'Hoker et al showed that the general SUSY solution of this type involves a genus g Riemann surface Σ, and is determined by two harmonic functions on this Riemann surface. These functions can have singularities which give AdS_5 x S^5 spikes.

- D’Hoker et al showed that another type of singularity is also allowed, corresponding to a 5-brane “throat” (surrounded by an S^3 instead of an S^5), involving D5-branes or NS5-branes wrapping AdS_4 x S^2.
- General non-singular solutions have two AdS_5 x S^5 spikes, and various 5-brane “throats”, interpreted as near-horizon of D3-branes intersecting 5-branes (with generally different ranks and gauge couplings) :





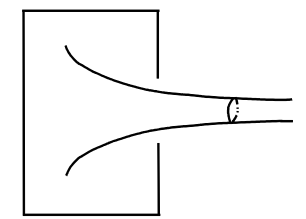
- The parameters of these solutions include the two asymptotic 5-form fluxes = the ranks N_1, N_2. We showed that one can take the limit of N_2 going to zero and obtain a smooth geometry with a single $AdS_5 \times S^5$ region. This is dual to the $SU(N_1)$ theory on AdS_4 (or a half-line), as we wanted !
- Boundary of boundary not an issue in 10d. Global symmetries realized by gauge fields on 5-branes.



- Discrete parameters : n = number of NS5-brane stacks, m = number of D5-brane stacks ($n+m>0$).
- For each 5-brane stack there are 2 additional continuous (integer after quantization) parameters. Can think of them as the number of 5-branes in each stack, and the number of D3-branes that end on each of these 5-branes. More precisely, when have both D5s and NS5s the latter is replaced by the linking numbers. The 5-form flux in each throat is ambiguous (since $dF_5 = F_3 \wedge H_3$, no unique conserved 5-form), related to ambiguity in definition of linking numbers.

- Exact match of SUGRA parameters to classification of field theory boundary conditions. Solutions are explicit (elementary functions) !
- 5-branes separated geometrically according to linking numbers ~ bending, as expected.

(a)

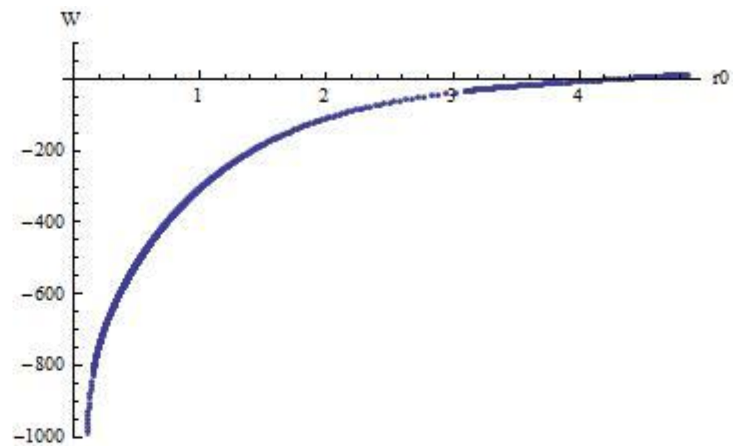


- Solutions are weakly curved (except deep inside the 5-brane "throats") as long as all the numbers of 5-branes and D3-branes are large. Natural to scale $N_5 \sim N^{1/2}$ for large N to get fixed geometry at large N, but not necessary.

Confinement in $\mathcal{N}$=4 SYM !?

- $\mathcal{N}$=4 SYM has many nice properties, but usually confinement is not one of them…
- One way to get confinement from $\mathcal{N}$=4 SYM is to add masses, giving the $\mathcal{N}$=1* theory that has confining vacua. Putting the theory on AdS_4 leads to a very similar structure (but with 16 supercharges). As we already saw, classical SUSY vacua involve scalar VEVs with $X_i = \tau_i / L_{AdS}$, and in particular (for D3-branes ending on a single D5-brane) can break SU(N) completely. This gives screening of electric charges, and confinement of magnetic charges.

- The S-dual vacuum, corresponding to D3-branes ending on an NS5-brane, thus has a magnetic condensate leading to screening of magnetic charges and confinement of electric charges ! Thus the $\mathcal{N}=4$ SYM theory on AdS_4 in this vacuum confines, at least for $g_{YM}>>1$.
- Using our gravitational solutions we can analyze this theory also in the limit of strong 't Hooft coupling, and we find that it also confines, with a string tension $T \sim g_{YM}^2 N/L_{AdS}^2$; we can now holographically compute the quark-anti-quark force :



- Interesting for understanding confinement in AdS_4… (with Berkooz, Tong, Yankielowicz)

Summary

- Found holographic duals for $\mathcal{N}=4$ SYM on AdS_4, with various boundary conditions preserving maximal supersymmetry.
- Should now compute in these solutions ! In particular, compute full spectrum holographically (from SUGRA, from D5-branes and NS5-branes and from stretched strings/branes).
- For D3-branes ending on 5-branes, so far we just computed the 1-point functions of some chiral operators (easy). Different from weak coupling.
- Many possible generalizations.
- Can this help us understand confinement in AdS_4 ?